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***Statistical Search for High Water
at Board Landing Bridge,
Truro, Nova Scotia***

G. Godin, P.A. Bolduc, D.G. Mitchell and S. Yuen

Marine Sciences and Information Directorate
Department of Fisheries and Oceans
Ottawa, Ontario

1981



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STATISTICAL SEARCH FOR HIGH WATER
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1. ABSTRACT

A small set of intermittent tidal observations on the height of high water at the Board Landing Bridge is scrutinized in order to estimate a height for the highest high water likely to occur there. The random portion of the signal is evaluated following a regression analysis with Saint John and other stations. The predicted height of high water at Saint John gives the best fit, suggesting little coupling between nontidal events at the two sites. A height of 33.3 ft with a return period of about 90 years is inferred.

RÉSUMÉ

Un petit nombre d'observations intermittentes sur la hauteur de la pleine mer au site du pont Board Landing est étudié afin d'estimer la hauteur de la plus forte pleine mer qui pourrait y être sentie. On évalue la portion aléatoire du signal à l'aide d'une analyse de régression avec Saint Jean et d'autres stations. La hauteur prédite de la pleine mer à Saint Jean donne les meilleurs résultats impliquant qu'il y a peu de corrélation entre les événements autres que la marée aux deux sites. On infère une hauteur de 33.3 pieds avec une période de retour de 90 ans.



Board Landing Bridge, Tidal Bore Road, Truro County, Nova Scotia (FGB photo, 1981).

2. INTRODUCTION

In an effort to obtain an estimate of the height of the high water at Board Landing Bridge (hereafter BLB) which has little probability of being exceeded, we carried out a statistical search of the existing, but small (two years only) body of tidal observations there (frontispiece). Strong shallow-water distortion in the area precluded the use of a direct harmonic analysis, so that we calculated correlations between the time of travel of high water to BLB and its height there, against the height of high water at a number of nearby "reference" stations as shown in the following:

Saint John (observed and predicted time and height of high water),
 Chignecto (predicted, using 81 days of observations),
 Grindstone (predicted from 87 days),
 Minas (predicted from 86 days) and
 Cobequid (predicted from 87 days).

Regressions were also made between the observed height and time of high and low water at Minas and Cobequid against Saint John. The results of previous investigations are summarized in the report C.D. 77.3.9-1 compiled by DILLON (Consulting Engineers and Planners): Tides in the Cobequid Bay and the Salmon River Estuary.

We have long series of reliable observations for Saint John but its distance from BLB suggested that we test the other sites as references, e.g. the submerged-gauge stations set up in the upper Bay of Fundy during 1976. These are more representative of the tide in the basin and in particular Cobequid monitors the wave just as it enters Salmon River. (The submerged gauges were not in operation during 1971-2). Harmonic constants were obtained from the observations; these being almost noise-free we could obtain reliable estimates of all the major constituents by superresolution. Predictions were prepared for the time interval of interest.

We retained only Saint John eventually because the observed extremes fell close to the regression curve, while those for the submerged stations strayed. Weakness in the predictions for the submerged stations and irregularities in the original recording at the Cobequid site were the probable causes for this erratic behaviour.

Once Saint John was retained and the regression verified, we checked to determine if there was strong coupling between events of nontidal origin at Saint John and BLB. The regression with the predicted high water at Saint John gave slightly better results than with the observed high water; this suggests weak coupling between the two sites for nontidal events. An additional inspection of extreme tides both at Saint John and BLB suggests also that deviations from the predicted tide at Saint John are not strongly correlated with deviations at BLB from values predicted from the regression formula.

We conclude that we may obtain an estimate of the height of high water

at BLB from the predicted high water at Saint John and the local extreme from the observed deviations from the regression. This was done and is presented in a later section.

The tide in the Bay of Fundy is of the perigean type (larger when the moon is nearer, i.e. every 27.55 days) because the frequency of the component, N_2 , (created by the variation in the moon's distance) falls very near the resonant frequency of the bay (Godin 1980). The major lunar and solar components M_2 and S_2 are also amplified; their interference creates spring and neap tides. Whenever spring tides coincide with perigean tides, the largest tides of the year occur in the Bay of Fundy. This corresponds to the astronomical situation in which the moon is in perigee during new or full moon.

The resonance conditions are different inside Minas Basin: M_2 is further amplified, while N_2 is nearly constant. The dynamic response of the main body of the Bay of Fundy and of Minas Basin to the same impulse must be quite different. Some chance coupling between the two on the other hand might create a dramatic response of the whole system. Friction becomes important in the upper reaches of the bay and the tide takes eventually the form of a free travelling wave increasingly distorted by friction and is eventually stopped by it.

Only high water is felt at BLB as the water during ebb recedes a few kilometers away. The tide seen there is the last breath of a once young and healthy signal passing by Saint John. It may easily be perturbed by meteorological conditions and the records of high water should be fairly irregular even if accumulated with the greatest care. In the absence of events of meteorological origin, the worst conditions for flooding will be created when perigean and spring tides coincide and the river is undergoing a maximum discharge. Higher levels downstream impede the flow and create higher level upstream even where no tide is felt.

3. DATA PROCESSING

The basic data consist of discontinuous hourly observations on the height of the water level at BLB taken by Water Survey of Canada during some months of 1971 and 1972. The observations cover the approximate time of high water although they do miss it occasionally. We applied Lagrangian interpolation to the data in order to extract the time and height of local high water; using the material available and rejecting unuseable data, we were left with 315 times and heights of high water, these forming a more or less adequate sample for further investigation.

We wish to represent the height, H , of the high water at BLB and the time, Δt , it took to reach from the reference station in terms of the height of high water at the reference station by:

$$\hat{y} = a + b\hat{x}$$

where \hat{y} is the height (feet) of high water at BLB or the time (hours) it took high water to reach it, a the y intercept, b the regression coefficient and \hat{x} the height of high water at the reference station (predicted or observed).

This is a least square fit and one needs some statistics to evaluate its representativeness.

Variable x: mean \bar{x} standard deviation s_x

Variable y: mean \bar{y} standard deviation s_y

The standard error of the estimate s_E :

$$s_E = \sqrt{\frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n-2}}$$

where y_i is the actual value of the i^{th} sample and \hat{y}_i is its value deduced from the regression. It is a measure of the goodness of fit between the assumed straight line relation and the actual data.

The confidence interval of the regression coefficient is:

$$\pm t_{[\frac{1}{2}(1+g)), n-2]} s_b \quad \text{at } 100g\% \text{ confidence level.}$$

Choosing 99% implies $g = .99$. This gives $\frac{1}{2}(1+g) = .995$ $n-2 = 313$, so that $t_{[.995, 313]} \sim 2.59$ from the student t distribution.

$$s_b = \frac{s_E}{\sqrt{\sum_{i=1}^n x_i^2}}$$

The reduction in the sum of squares due to the regression is:

$$R^2 = \bar{ny}^2 + b \sum_{i=1}^n x_i y_i$$

The % of variation in the sum of squares contributed by the regression is:

$$\frac{R^2}{\sum_i y_i^2} \times 100$$

The residual sum of squares is $\sum_i (y_i - \hat{y}_i)^2$ and its mean values is s_E^2 . The hypothesis of 0 regression is tested with the t test and the F test. The t test consists in forming the variable:

$$t = \frac{b - B}{s_b}$$

where b is the calculated regression coefficient and B is the true coefficient hypothesized to be 0. The hypothesis is false at 100g% probability if:

$$t \equiv \frac{b}{s_b} > t_{\frac{1}{2}(1+g), n-2}$$

at 99% $t_{.995, 313} \sim 2.59$

at 99.9% $t_{.9995, 313} \sim 3.32$

The F test consists in forming:

$$F = \frac{\text{mean square explained by the regression}}{\text{residual mean squares}}$$

The hypothesis of 0 regression implies $F = 1$. The hypothesis is rejected if at 100g% probability:

$$F > F_{g, 1, n-2}$$

at 99% $F_{.99, 1, 313} \sim 6.63$

at 99.9% $F_{.9995, 1, 313} \sim 12.5$

In the plots, the limits of prediction are given by the curves:

$$\begin{cases} U \\ L \end{cases} = \bar{y} \pm t_{\frac{1}{2}(1+g), n-2} s_{\bar{y}}$$

at 100g% confidence, where:

$$s_{\bar{y}} = s_E \sqrt{1 + \frac{1}{n} + \frac{(\bar{x} - \bar{x})^2}{\sum_i x_i^2}}$$

\bar{x} being a regular sequence of values along the abscissa. The printouts in the appendix give all the statistics mentioned:

variable 2	x :	average value \bar{x}	standard deviation s_x
variable 1	y :	average value \bar{y}	standard deviation s_y

the correlation coefficient between the two variables:

$$r = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2 \sum_i (y_i - \bar{y})^2}}$$

the number of samples, n ,
 the regression coefficient, b ,
 the standard error of the regression coefficient:

$$\frac{s_E}{\sqrt{n} s_x}$$

the computed t value b/s_b for the t test,
 the proportion of variation:

$$R^2 \times 100 / \sum_i y_i^2$$

the residual sum of squares and its mean values s_E^2
 the y intercept, a ,
 the standard error of the estimate, s_E ,
 the F level, mean square explained by the regression/residual
 mean square,
 the contribution of the regression to the sum of squares
 (1 degree of freedom) and
 the residual sum of squares.

We see from Appendices 4 and 5, that all the regressions with the chosen reference stations pass the t and F tests to a high degree of reliability; therefore any one of the stations chosen is suitable as a reference station. The inspection of the results will indicate which one is optimum. The results of the statistics are summarized in Table 1.

Table 1 indicates that Saint John is the best predictor for the heights. The correlation for the pure tide as input (Saint John predicted) is a shade higher than for the actual level recorded (Saint John observed). The difference (.004) in the correlation coefficient is not statistically significant but an inspection of the scatter diagrams (Figure 1) indicates that Saint John (predicted) contains fewer outliers in the region of extreme levels than Saint John (observed). We conclude that the predicted tide at Saint John (Saint John (p)) is a better predictor of the height of high water at BLB. The weaker correlations for the submerged stations are mainly due to the fact that shallow-water distortions become appreciable in Minas Basin (Minas and Cobequid) and that standard harmonic predictions fail to predict adequately the height of extremes. The better correlations obtained for stations in the Chignecto arm of the bay (Grindstone and Chignecto) suggest that the harmonic analysis of these records has been more successful in reproducing the levels actually observed. All of these can be discarded as predictors of the height of high water at BLB, including Cobequid the nearest available station to BLB.

The coefficient of regression on the times (Figure 2) is negative in all cases (see also printouts in the appendix and Table 1), indicating that the velocity of the free wave reaching BLB increases with the height of high water. The correlation values follow our intuition this time, Cobequid appearing as the best predictor. We obtain a few seemingly absurd values for Cobequid: some points indicate the tide reaches BLB before Cobequid. Since these latter values are of the order of 0.1 to 0.2 hours, they fall within

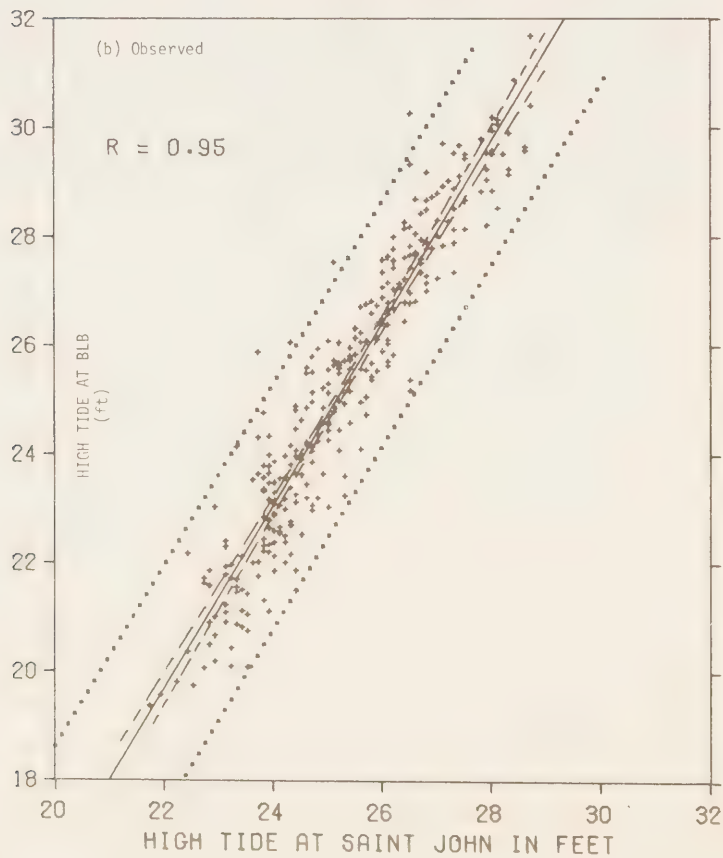
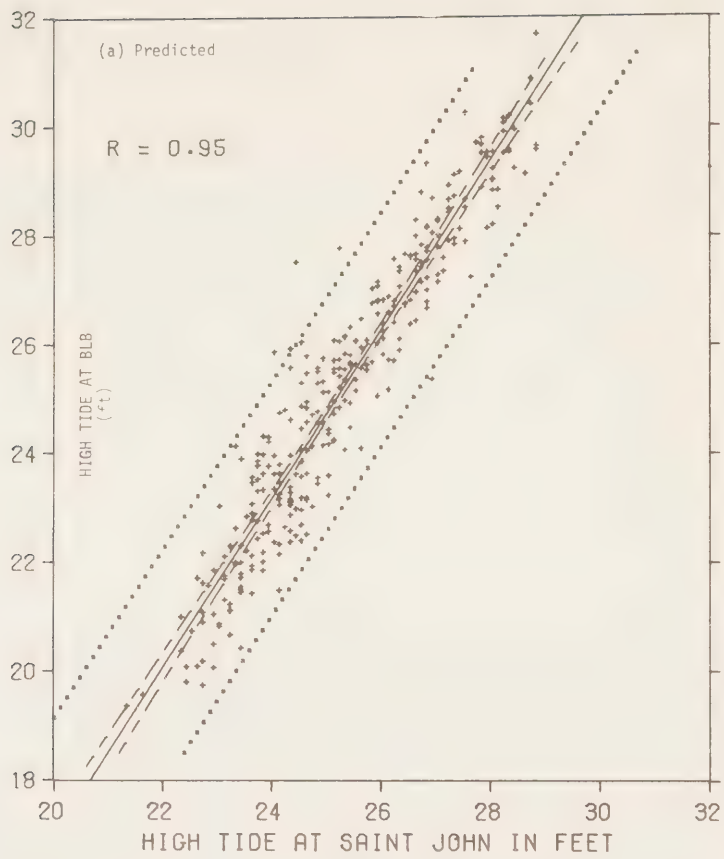
Table 1. Summary of the regression calculations arranged in decreasing order of correlation with respect to the heights.

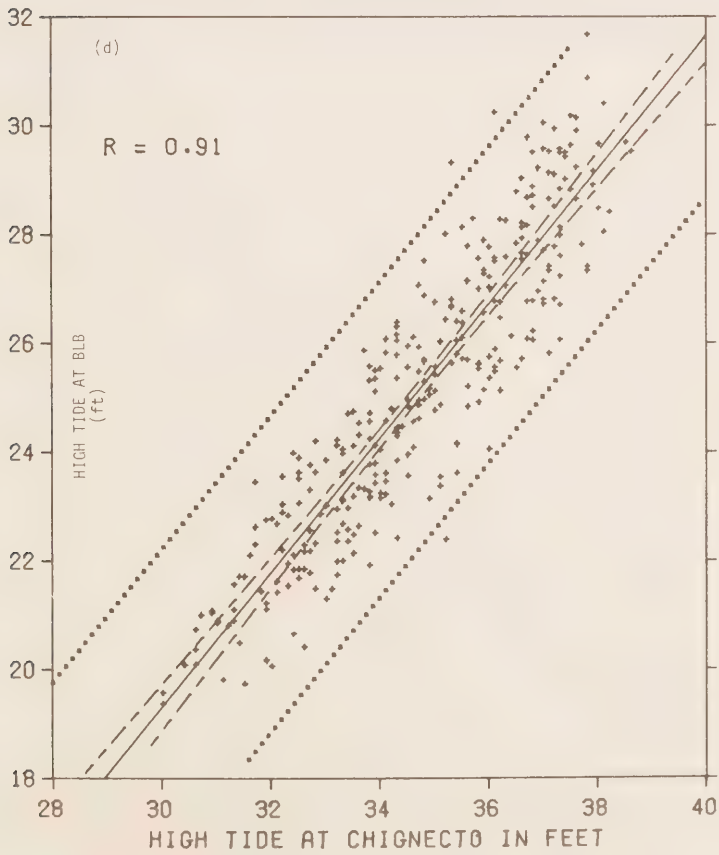
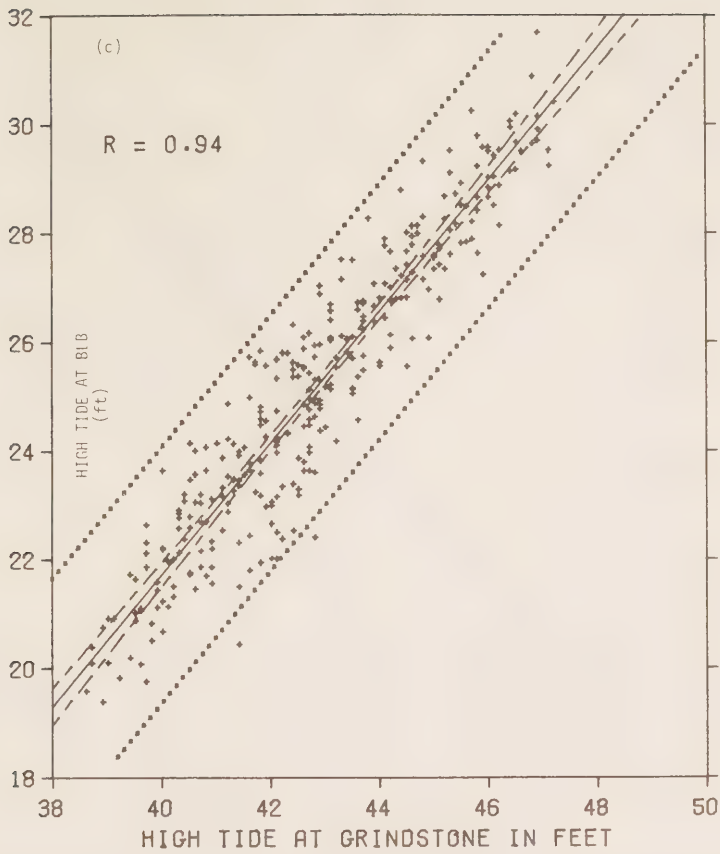
Reference Station	Height of High Water at BLB		Time Difference	
	Correlation r	Regression Equation H (ft)	Correlation r	Regression Equation Δt (hours)
Saint John (predicted)	.950	$H = -14.024 + 1.550H_{SJ}^{pred}$	-.247	$\Delta t = 2.321 - .030H_{SJ}^{pred}$
Saint John (observed)	.946	$H = -17.126 + 1.673H_{SJ}^{obs}$	-.235	$\Delta t = 2.295 - .030H_{SJ}^{obs}$
Grindstone (predicted)	.941	$H = -26.611 + 1.208H_G$	-.481	$\Delta t = 3.562 - .053H_G$
Chignecto (predicted)	.908	$H = -17.689 + 1.233H_C$	-.443	$\Delta t = 3.458 - .062H_C$
Cobequid (predicted)	.901	$H = -29.980 + 1.073H_{Cob}$	-.650	$\Delta t = 4.490 - .079H_{Cob}$
Minas (predicted)	.877	$H = -30.291 + 1.184H_M$	-.632	$\Delta t = 4.519 - .084H_M$

Table 2. Time necessary for high water to reach BLB for various values of the height of high water at the reference station using the regression relation.

Reference Station	Height of High Water at the Reference	Time Needed for High Water to Reach the Site
	(ft)	Δt (h)
Saint John	22	1.64
	24	1.58
	26	1.53
	28	1.47
	30	1.41
	32	1.35
Minas	43	.93
	45	.77
	47	.60
	49	.43
	51	.27
	53	.10
Cobequid	47	.78
	49	.62
	51	.46
	53	.30
	55	.15
	57	-.01

Fig. 1. Regressions between predicted or observed heights at stations in the area and at BLB. The stars are the samples, the solid line is the regression straight line and the dashed lines are the limits of the regression straight line at 99% of the samples. The dotted lines should contain 99% of the samples. The coefficient of correlation is given in the upper left corner. a) Between the predicted height of high water at Saint John and the observed height of high water at BLB. b) Between the recorded height of high water at Saint John and the observed height of high water at BLB. Note the samples fall close to the regression curve in the region of extremes for the predicted height at Saint John and the more even distribution of stray points for Saint John predicted. c) Between the predicted height of high water at Grindstone and the height of high water at BLB. d) Between Chignecto and BLB. e) Between Cobequid and BLB. f) Between Minas and BLB.





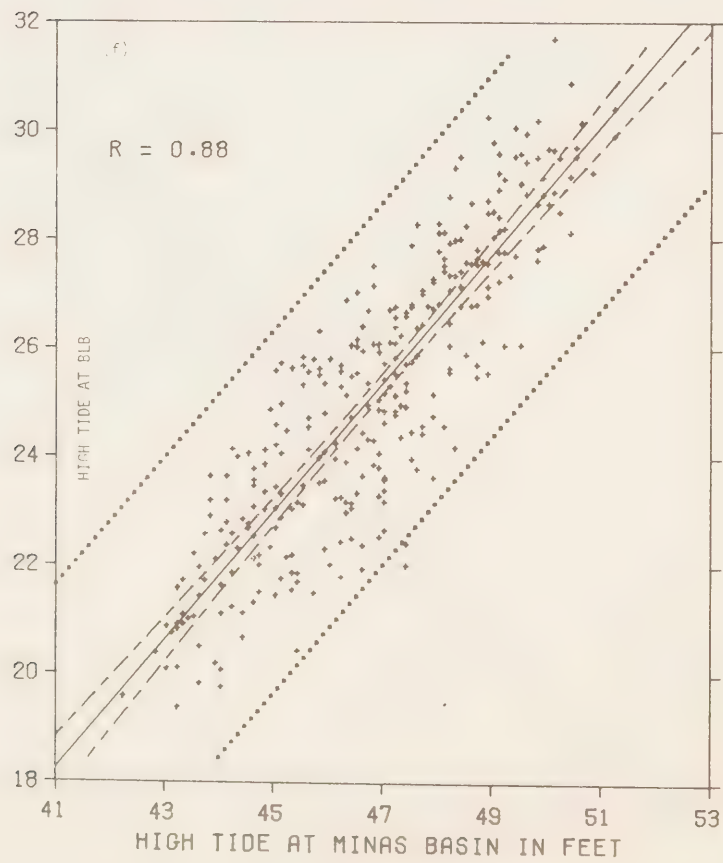
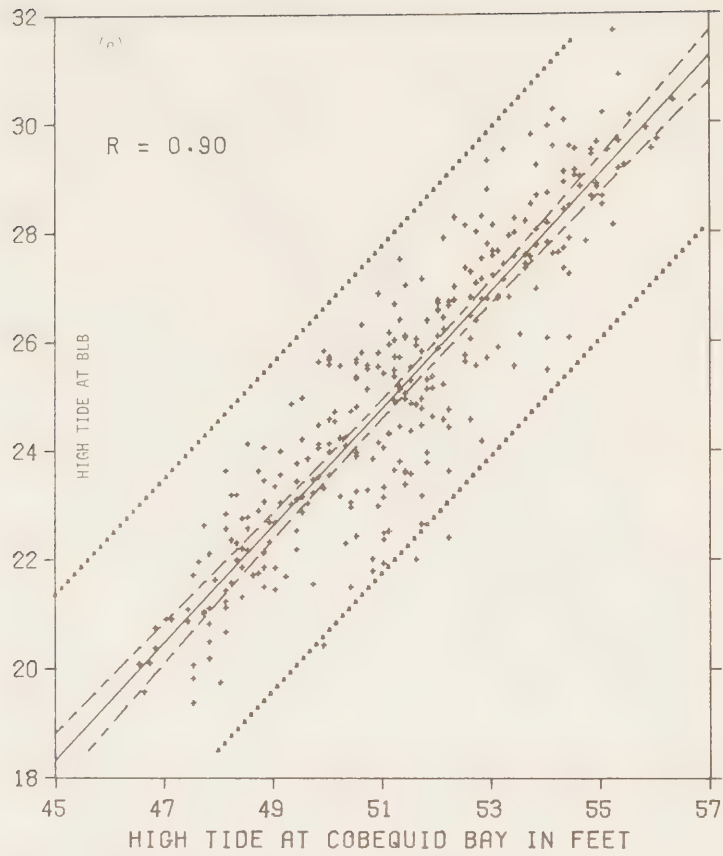
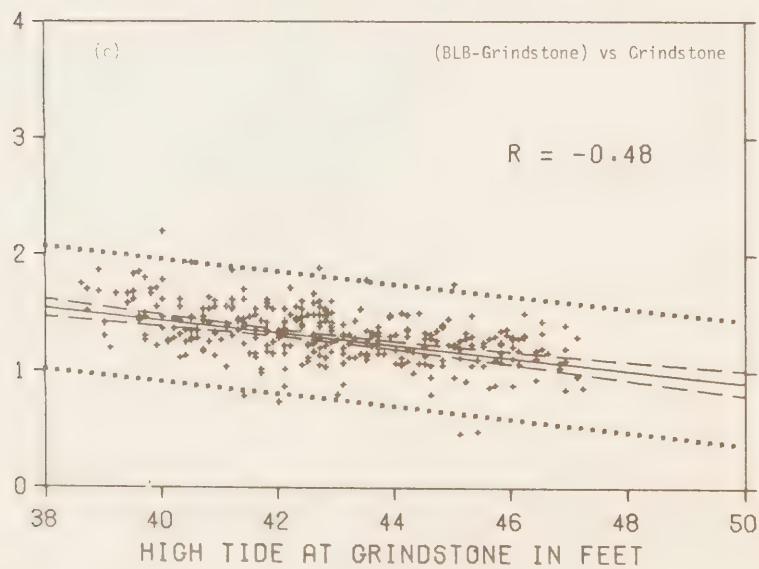
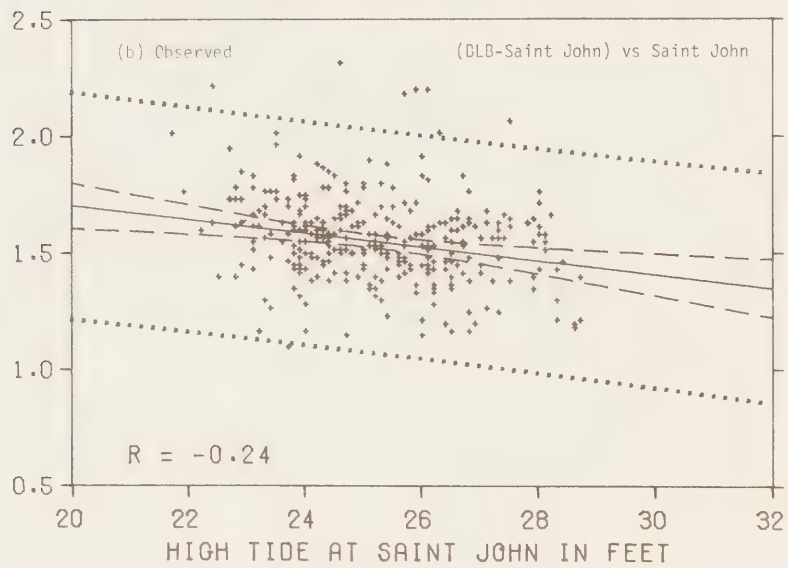
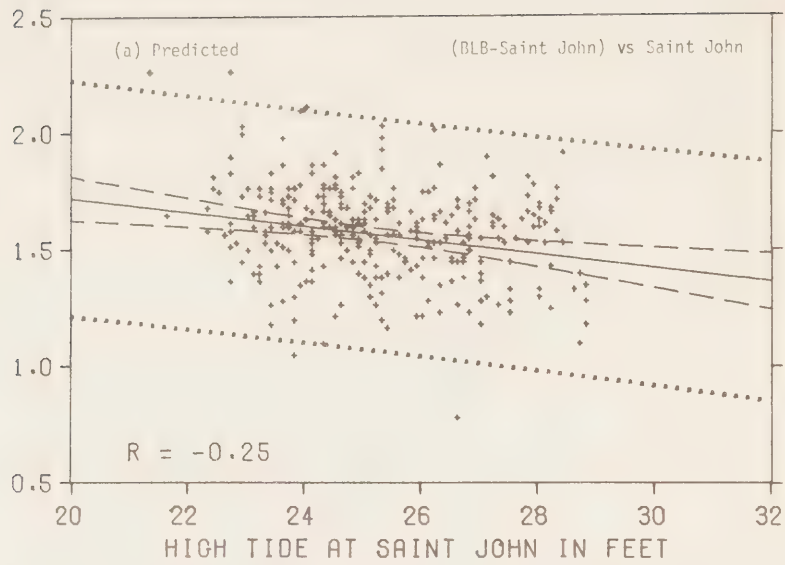
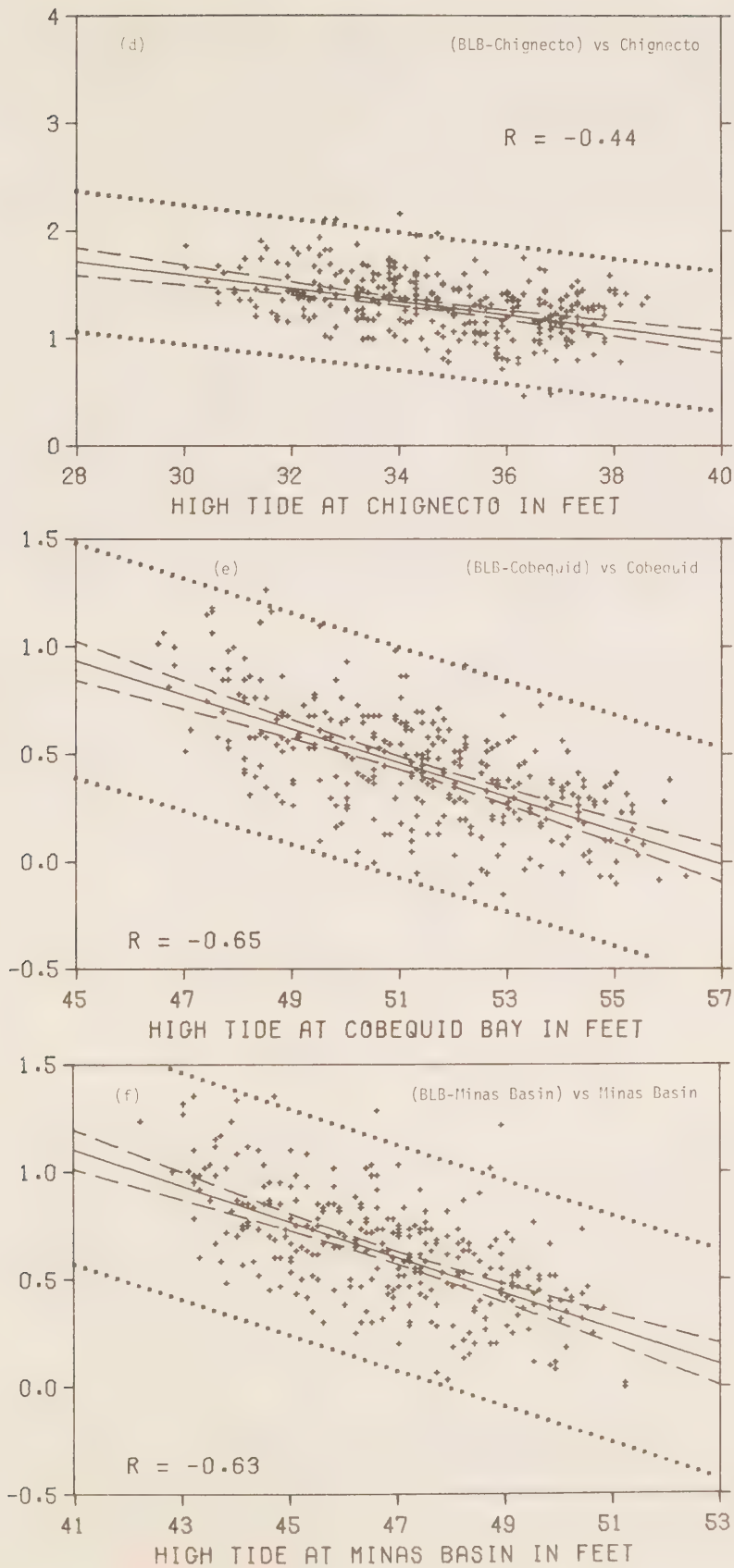


Fig. 2. Regressions between the predicted or observed height of high water at stations in the area and the time difference between the occurrence of high water at the particular location and BLB. A negative slope indicates that the higher the high water the faster it progresses toward Salmon River, i.e. toward BLB. a) Between the predicted height of high water at Saint John and BLB. b) Between the observed height of high water at Saint John and the time interval. Note again the use of Saint John predicted as input results in a more natural distribution of sample points. c) Regression with Grindstone. d) Regression with Chignecto. e) Regression with Cobequid. f) Regression with Minas.





the limits of tolerance of the predictions we can prepare for the data at Cobequid. We calculate (Table 2) the mean time it takes for the high water to reach BLB and the velocity of the tidal wave for various heights of high water at the reference stations.

Table 2 makes it quite clear that there might be dramatic changes in the velocity of progress of the tidal wave during very high tides.

4. SEARCH FOR HEIGHT OF HIGHEST HIGH WATER

The high water reached at BLB is the superposition of a pure tide signal and of an additional displacement contributed by the response of Minas Basin to local winds and pressure fields, and to disturbances coming from the Bay of Fundy. The height of high water due to the tide has a mean and standard deviation but it definitely is not a random variable: it has a well-defined upper bound determined by the combination of the tidal constituents producing the largest possible tide. The disturbance in level may be treated as a random variable whose upper bound can be ascertained at a given confidence level with the help of statistics.

We cannot dissociate directly the pure tide at BLB from other perturbations, but we may attempt to do so with the help of the statistics just obtained from comparisons with Saint John. The question is: is a disturbance at Saint John automatically reflected in the level at BLB? We therefore review extreme events which can instruct us about the association or lack of association between nontidal events at Saint John and BLB:

- a) levels exceeding 30 ft at BLB,
- b) extreme high tides recorded at Saint John during 1971-72 and their repercussion at BLB, and
- c) the extreme level recorded at Saint John.

4.1 Scrutiny of extreme levels at BLB and Saint John

We go over the levels exceeding 30 feet at BLB in order to determine whether:

- a) they differ appreciably from the regressed values and
- b) if so, whether Saint John was disturbed during the event and in what way?

Table 3 gives the observed level at BLB, the regressed level using Saint John predicted, their difference, the regressed level using Saint John observed, their difference, the time and height of the predicted high water at Saint John, the time and height of the observed high water at Saint John and finally the height difference between the predicted and observed high water. The last value indicates whether Saint John was perturbed or not during the high level at BLB.

Table 3 suggests that although the extreme of 31.6 ft at BLB seems at first as the most noteworthy, the later extreme occurring at 14 hours involves an even larger local disturbance. Three events appear to reflect a true disturbance inside Minas Basin: the one at 15 hr 7 October, 1971 and the two just mentioned which occurred 15 April, 1972. The tide at Saint John was unperturbed during the first event. On the 15 April 1972 the first big tide at Saint John was completely normal while the level in Minas Basin was significantly abnormal; we have a situation in which a strong but normal tide at Saint John is associated with a disturbance in Minas Basin. Saint John was significantly perturbed at 14 hours: its level was significantly lowered while the level at BLB was abnormally raised.

Table 3. Levels exceeding 30 ft at BLB in regards to the possible occurrence of events at Saint John.

Time and Date (h, d, m, y)	Level Observed (ft)	Regressed Value			Tide at Saint John			Height		
		Saint John H (ft)	Pred.Diff. Δ	Saint John H (ft)	Obs.Diff. Δ	Observed Time	Observed Height (ft)	Predicted Time	Predicted Height (ft)	Difference Δ (ft)
1.0 26/04/71	30.4	30.0	.4	30.9	-.5	23 48	28.7	23 50	28.4	.3
14.9 07/10/71	30.1	29.2	.9	29.9	.2	13 22	28.1	13 20	27.9	.2
12.8 03/11/71	30.8	30.6	.2	30.4	.4	11 21	28.4	11 20	28.8	-.4
.9 14/04/72	30.1	29.7	.4	29.7	-.4	23 20	28.0	23 20	28.2 ^{on the 13th}	-.2
1.5 15/04/72	31.6	30.5	1.1	30.9	.7	00 07	28.7	00 05	28.7	0
14.1 15/04/72	30.2	28.6	1.6	27.2	3.0	12 39	26.5	12 35	27.5	-1.0
14.0 24/10/72	30.0	29.7	.3	29.9	.1	12 26	28.1	12 25	28.2	-.1

We now review the extreme tides at Saint John during 1971 and 1972 and present (Table 4) the extreme high tides observed at Saint John and the corresponding observed levels at BLB. The table suggests that during these extreme tides, things were regular at Saint John with the exception of 21 and 22 November 1972 when the level was somewhat depressed. We detect a significant abnormality at BLB on the 4 November 1971, 15 April 1972 and 12, 13, 15 May 1972. The height was lower than expected in three events and it was higher in one, while in the mean time, everything seemed to be going fine at Saint John.

We consider finally the extreme level at Saint John. Our records indicate that the highest instantaneous level at Saint John is: 30.25 ft. It occurred at 00 h 15 min 6 April 1977; the predicted high tide was 00 h 45 min and 28.1 ft. The actual high tide therefore was 30 min early and 2.1 ft higher. This must have been created by a major disturbance since, as we have seen, the tide at Saint John comes like clockwork. The Bay of Fundy appears

Table 4. Search for coincident abnormalities in the level at Saint John and at BLB, and in the time taken by high water to reach BLB for extreme tides at Saint John predicted for the years 1971 and 1972.

Date	SAINT JOHN										BLB			
	Time					Time					Height			
	Observed Time	Observed Height (ft)	Predicted Time	Predicted Height (ft)	Difference in Height (ft)	Time of High Water T (h)	Time Taken to Progress from Saint John Δt (h)	Regressed Value Using Saint John Pred. Δt (h)	Difference (h)	Height Observed Saint John (ft)	Regressed Height Saint John Pred. H_P (ft)	Difference (ft)	Height Observed Saint John (ft)	Difference (ft)
1971														
April 24	23 00	28.7	23 05	28.3	.4	x								
25	23 48	28.7	23 50	28.4	.4	x								
Oct 5	11 47	28.1	11 40	28.5	-.4	x								
6	12 38	28.5	12 30	28.5	0	x								
Nov 2	10 35	28.0	10 30	28.5	-.5	12.0	1.42	1.47	-.07	29.5	30.2	-.7	29.7	-.2
3	11 21	28.4	11 20	28.8	-.4	12.8	1.45	1.46	-.01	30.8	30.6	.2	30.4	.4
4	12 11	28.3	12 10	28.5	-.2	13.6	1.46	1.47	-.01	29.2	30.2	-1.0	30.2	-1.0
1972														
April 13	23 20	28.0	23 20	28.2	-.2	24.9	1.54	1.48	.08	30.1	29.7	.4	29.7	.4
15	00 07	28.7	00 05	28.7	0	1.5	1.40	1.46	-.06	31.6	30.5	1.1	30.9	.7
May 12	22 56	28.6	22 55	28.5	.1	24.1	1.17	1.47	-.30	29.5	30.2	-.7	30.7	-1.2
13	23 49	28.8	23 45	28.8	-.2	25.0	1.14	1.46	-.28	29.6	30.6	-1.0	31.1	-1.5
15	00 42	28.3	00 40	28.6	-.3	1.9	1.20	1.46	-.26	29.1	30.3	-1.2	30.2	-1.1
Oct 23	11 40	27.9	11 40	28.1	-.2	13.2	1.53	1.48	.05	29.5	29.5	0	29.5	0
24	12 26	28.1	12 25	28.2	-.1	14.0	1.57	1.48	.09	30.0	29.7	.3	29.9	.1
Nov 21	11 18	28.0	11 20	28.5	-.5	13.0	1.70	1.47	.23	29.5	30.2	-.7	29.7	-.2
22	12 09	28.0	12 10	28.6	-.6	13.9	1.75	1.46	.29	29.9	30.3	-.4	29.7	.2

as a finely tuned system which responds most selectively to an applied disturbance; its response to the proper input is bound to be spectacular.

We wish to investigate if this level was created by local conditions and if it can be used to calculate an extreme height at BLB. The only local source of disturbance at Saint John which is abutting directly against the Bay of Fundy is the Saint John River; we must check if it was abnormally raised thus causing the abnormal reading. We list (Table 5) the observed and predicted height of high water before and after the event.

Table 5. Observed and predicted height of high water before and after the highest level ever recorded at Saint John.

Date 1977	Observed (ft)	Predicted (ft)	Difference (ft)
April 3	27.5	26.5	1.0
	27.4	27.1	.3
April 4	26.5	27.1	-.6
	27.7	27.8	-.1
April 5	28.1	27.4	.9
April 6	30.0*	28.1	1.9
	27.9	27.3	.6
April 7	28.1	28.1	0
	26.5	26.8	-.3

*smoothed hourly value

Flood conditions would have caused a persistent rise in levels during at least a few days; the abnormally raised levels lasted some 36 hours. This inclines us to the notion that the event took place in the Bay of Fundy. The weather records indicate moderate winds and somewhat rough seas but nothing of a hurricane nature. The response of the Bay of Fundy to such mild stimulus to create this unusual event deserves to be investigated further. We have no records at BLB for that date.

5. HEIGHT OF THE HIGHEST HIGH WATER USING THE EXTREME LEVEL AT SAINT JOHN

Using conventional regression statistics the maximum height for BLB is:

$$y_d = \hat{y}(30.25) + t s_y$$

where the value of t is determined by the level of confidence chosen and the number of samples. For 315 samples (313 degrees of freedom) and levels of 99.9 and 99%,

$$t = 3.32 \text{ and } 2.59$$

This gives:

$$\begin{aligned} y_d &= 35.7 \text{ ft, } 35.1 \text{ ft using the prediction} \\ &\quad \text{regression, and} \\ &= 36.3 \text{ ft, } 35.6 \text{ ft using the observed} \\ &\quad \text{regression} \end{aligned}$$

at 99% and 99.9.

These values are based on the assumption that all the samples have equal probability. In fact the extreme events are determined by a compound probability: we need a high tide *and* an abnormally high disturbance in the level. We can evaluate the probability of high water exceeding a given level at Saint John and we can study the residues from the regression to determine their probability distribution and the return period of the extreme. In this way we attempt to calculate heights corresponding to given return periods.

5.1 Tides of 29 ft and over at Saint John

As derived from analyses of annual records of level, the amplitude of the major constituents of the tide at Saint John are:

M_2	9.94 ft
S_2	1.63 ft
N_2	2.05 ft
O_1	0.37 ft and
K_1	0.50 ft.

In addition there are a larger number of constituents of lesser importance. The highest tide would occur if they were all in phase: this is never the case. We may assume that generally the smaller constituents cancel each other and the larger constituents may reinforce or weaken each other.

Perigee tides occur when M_2 and N_2 are in phase; spring tides when M_2 and S_2 are in phase. If the three are in phase, we have a coincidence of perigee and spring tides. The reference level being 14.56 ft, perigee tides would give a high water of 26.6 ft and spring tides, a high water of 26.1 ft on average. Spring perigee tides would give 28.2 ft. Because of the diurnal inequality, K_1 and O_1 increase one of the high waters and decrease the subsequent one; this would give a higher high water of 29.1 ft during spring tides, which should be a very rare event and one whose probability of occurrence we should investigate. Tidal forces have cycles of a month, a year, 9 years, 18 years and 26,000 years; the 26,000-year cycle is too long to be

perceptible in our records, but the 9-and 18-year cycles are certainly noticeable. Astronomy instructs us that the semidiurnal forces had an 18 year maximum in 1960 and 1978; the next peak is due in 1997 (Anonymous 1967). It will coincide with a minimum of the diurnal forces; therefore we cannot expect the largest tide ever to occur on that very year. If we inspect the tide tables for Saint John, we note predicted heights of high water of 29.1 ft on 4 April and 6 April 1958; the next highest high water is 29.1 ft 3 June 1977. Therefore the actual tide maximum in the Bay of Fundy occurs some one or two years before the peak in the semidiurnal forces and may be due in part to the accidental additional contribution of some of the lesser constituents.

In order to check on the interval of reoccurrence of extreme tides in the Bay of Fundy we prepared one hundred years of predictions covering the interval 1981 to 2080 for Saint John. We give (Table 6) the 100 highest tides arranged in chronological order and in magnitude (there are many more 28.7-ft tides than indicated since the computer was ordered to stop at the 100th entry). We notice that the absolute highest tide is indeed 29.1 ft and occurred 8 times in 100 years; one should be tempted to give it a probability of once every twelve years. But the tide being a deterministic signal, it follows the force applied and not the rules of dice. In fact we have two 29.1-ft tides on 6 and 8 May 2016 or twice in 24 hours, then another the following year on 26 May 2017. The next 29.1-ft tide occurs 17 May 2034; we therefore have four peak tides in 18 years. We note in passing that extreme tides in the Bay of Fundy occur either at midnight or at noon, the midnight tide occurring in spring and the noon tide occurring during autumn. Since both seasons are storm prone, we may expect an increased probability of catastrophic conditions during these extreme tides. We note also that there is a secular increase in the mean level along the Atlantic seaboard of 0.01 ft/year (Dohler and Ku 1970) which has not been taken into account because it is impossible to predict its persistence or lack of persistence in the future. Returning to our problem of extreme heights at BLB we recall that the observed extreme local level of 31.6 ft corresponded to a tide of 28.7 ft at Saint John. Therefore in the practical situation the existence of extreme local levels seems to be linked to large tides in the Bay of Fundy and not to specific extreme tides which exceed the average large tide by only a few tenths of a foot.

An example is the "Saxby Gale" which occurred 4 October 1869 (Hutchinson 1912; Crane 1969). Tide "hindcasting" for the day gives a high tide of 28.0 feet at 22:35 hours, a level which is more than one foot below the absolute highest tide. Nevertheless, around that time of that day, the Saint John area was subjected to one of the greatest storms and highest water levels recorded, if not in data records, at least in memory as being very destructive.

"The tide exceeded in height any previous record, the water dashing over all of the wharves, tearing vessels away from their moorings, wrecking them on the beaches, and damaging others by pounding them against wharves. The damage was not confined to Saint John. At Point Lepreau the bark Genil was

Table 6. One hundred maximum tides at Saint John between 1981 and 2080 showing date, time and maximum level (WLMAX) in feet. The list of j harmonic components used to prepare the table is given in appendix 1. The list was set up by studying the results of a succession of analyses and deducing what seemed the most plausible values of the constituents at Saint John. There is an element of subjectivity in this, especially in the case of L_2 and μ_2 which vary appreciably from year to year. The predictions obtained from this set will differ in some details from those found in tide tables for which the latest constituents are used as a base for predictions.

	DD/MM/YYYY	HH:MM	WLMAX		DD/MM/YYYY	HH:MM	WLMAX
001	04/05/1981	23:50	29.1	002	27/04/1994	00:10	29.1
003	05/11/1998	12:05	29.1	004	15/05/1999	23:45	29.1
005	06/05/2016	23:25	29.1	006	08/05/2016	00:05	29.1
007	26/05/2017	00:00	29.1	008	17/05/2034	23:50	29.1
009	06/05/1981	00:50	29.0	010	30/03/1998	00:45	29.0
011	17/05/1999	00:25	29.0	012	09/04/2016	00:35	29.0
013	15/11/2016	11:45	29.0	014	16/11/2016	12:40	29.0
015	19/04/2034	00:10	29.0	016	25/11/2034	11:20	29.0
017	20/04/2038	00:40	29.0	018	18/11/2051	11:00	29.0
019	01/05/2052	00:50	29.0	020	01/05/2056	00:20	29.0
021	01/11/2073	12:15	29.0	022	11/05/2074	23:55	29.0
023	13/05/2074	00:55	29.0	024	04/11/1994	11:35	28.9
025	26/04/1998	23:40	28.9	026	28/04/1998	00:35	28.9
027	04/11/1998	11:05	28.9	028	06/11/1998	12:50	28.9
029	06/05/2012	23:55	28.9	030	10/04/2016	01:15	28.9
031	31/03/2033	00:20	28.9	032	08/10/2033	11:55	28.9
033	06/11/2033	11:25	28.9	034	07/11/2033	12:20	28.9
035	20/04/2034	01:05	28.9	036	26/11/2034	12:15	28.9
037	05/06/2035	23:45	28.9	038	28/10/2038	12:10	28.9
039	11/04/2051	23:55	28.9	040	13/04/2051	00:55	28.9
041	10/05/2051	23:30	28.9	042	21/10/2051	12:20	28.9
043	19/11/2051	12:00	28.9	044	29/04/2052	23:50	28.9
045	28/05/2052	23:40	28.9	046	07/12/2052	12:00	28.9
047	08/11/2056	11:55	28.9	048	09/11/2056	12:30	28.9
049	25/03/2069	00:55	28.9	050	21/04/2069	23:30	28.9
051	23/04/2069	00:25	28.9	052	31/10/2069	12:00	28.9
053	01/11/2069	12:45	28.9	054	10/05/2070	23:40	28.9
055	12/05/2070	00:20	28.9	056	23/04/2073	00:05	28.9
057	31/10/2073	11:40	28.9	058	20/11/2074	12:20	28.9
059	07/04/1981	01:10	28.8	060	03/05/1981	23:10	28.8
061	02/06/1981	23:40	28.8	062	13/11/1981	12:15	28.8
063	05/11/1994	12:25	28.8	064	16/05/1995	00:05	28.8
065	09/02/1997	12:55	28.8	066	07/10/1998	12:15	28.8
067	24/11/1999	12:00	28.8	068	26/10/2011	11:15	28.8
069	08/05/2012	00:35	28.8	070	15/11/2012	12:05	28.8
071	21/03/2015	00:30	28.8	072	28/09/2015	12:00	28.8
073	27/10/2015	11:55	28.8	074	09/05/2016	01:05	28.8
075	17/10/2016	12:05	28.8	076	14/11/2016	10:50	28.8
077	09/04/2020	00:10	28.8	078	16/11/2020	12:05	28.8
079	06/11/2029	11:50	28.8	080	09/10/2033	12:30	28.8
081	16/05/2034	23:05	28.8	082	19/05/2034	00:45	28.8
083	28/10/2034	12:40	28.8	084	15/03/2051	01:00	28.8
085	30/05/2052	00:20	28.8	086	08/11/2052	12:15	28.8
087	08/12/2052	12:45	28.8	088	14/03/2055	12:20	28.8
089	13/04/2055	00:15	28.8	090	21/10/2055	11:50	28.8
091	12/10/2068	12:05	28.8	092	29/11/2069	11:40	28.8
093	10/06/2070	00:00	28.8	094	25/03/2073	00:20	28.8
095	13/04/2074	00:20	28.8	096	14/04/2074	01:00	28.8
097	12/11/1981	11:25	28.7	098	12/12/1981	12:00	28.7
099	23/05/1982	23:45	28.7	100	06/05/1985	00:05	28.7

wrecked and 11 lives lost. In Albert County the tides caused damage estimated at \$250,000, and Westmorland had the highest tide ever known, the water at Moncton rising six and a half feet above previous records. Buildings were blown away and smashed into bits." (Crane 1969).

Hutchinson (1912, p. 256) stipulated that low atmospheric pressures (barometer 29.3 inches of Hg) were recorded around that time. The combined effects of the storm surge and the tidal wave produced a level which exceeded the absolute highest tide level.

5.2 Study of residuals from the regression with Saint John

We give (in the appendix) the differences between the observed height at BLB and the regressed height (using the Saint John high water as the predictor) and also the error for the time of travel of high water. These discrepancies are given for the observed and the predicted values at Saint John. We consider these residues as representing the random portion of the height of high water sensed at BLB. Their distribution in time allows us to pick monthly extremes for the height deviations and establish statistics for their recurrence. Together with an assigned frequency of tides over 29 feet they will allow us to establish return periods for extreme heights. Also the inspection of the residues will indicate times during which the levels were abnormally high. Abnormally high levels occurred on the following dates:

28-29	July	1971
24-25	August	1971
15-16	April	1972
9-10	October	1972

Tests of randomness (white noise tests) of the residues are shown (Figures 3 and 4) and indicate that raised or depressed levels have a tendency to persist.

To establish a return period we selected monthly extremes in the height residues and obtained 15 sample extremes (Table 7). Without doubt they form a poor sample because many months were not complete and some of the high waters had not been observed during that month, but it is all we have.

We plotted these values on probability paper (Figure 5) for the predicted and observed heights at Saint John. The plot for the predicted Saint John (Fig. 5a) has an outlier, which makes problematic the slope of the straight line; the plot of the observations (Fig. 5b) is somewhat less

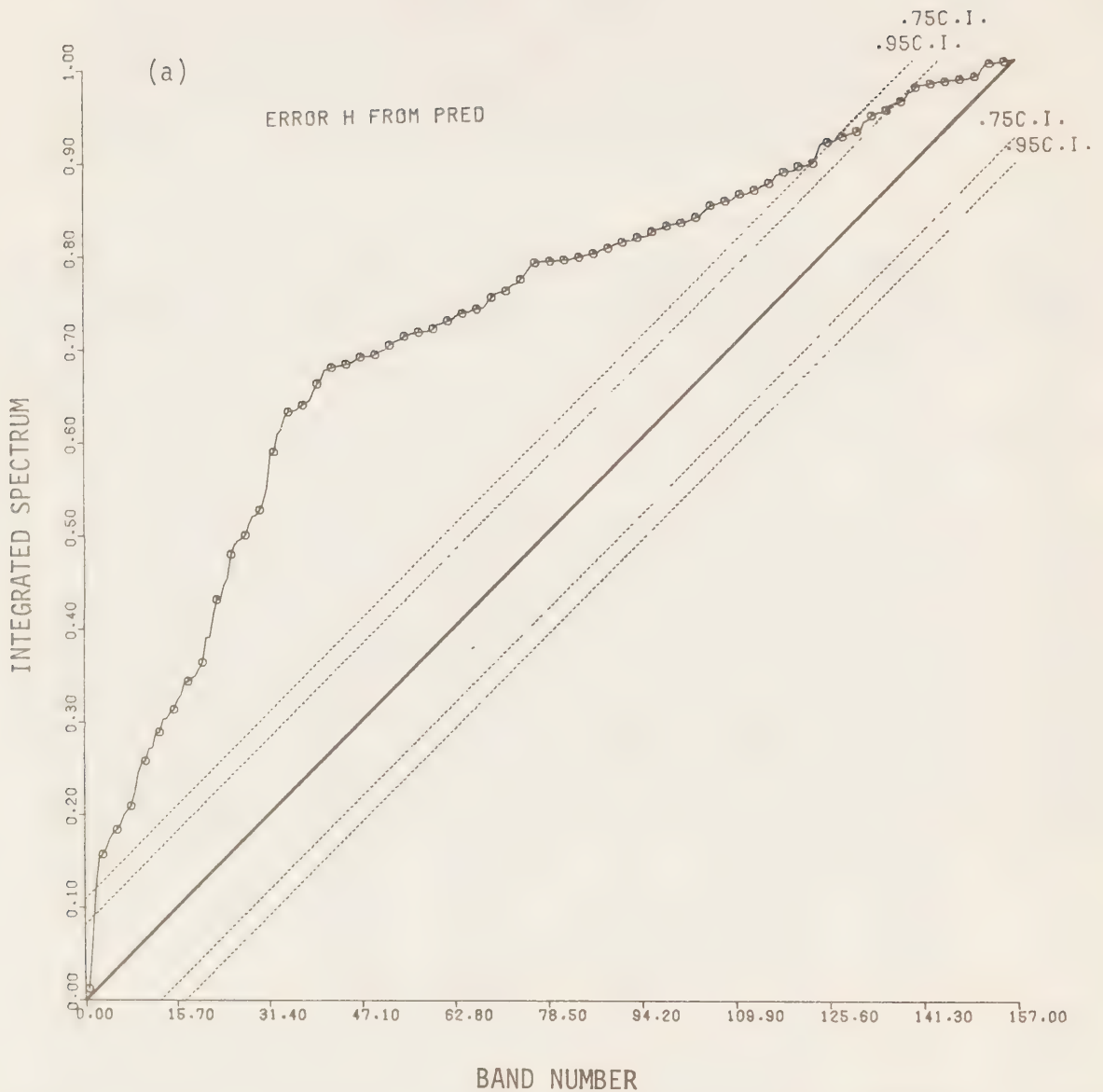


Fig. 3. White noise test on the residues obtained by regressing the predicted or observed height of high water against the height of high water at BLB. The spectrum of white noise should be flat and accumulated samples of its spectrum should fall along the solid straight line. The two dotted lines give the 75 and 95% confidence intervals for the test. It is evident that the spectrum strays from whiteness in the lower frequencies and returns towards whiteness in the higher frequencies. a) Predicted height of high water at Saint John against the height of high water at BLB. b) Observed height of high water at Saint John against the height of high water at BLB.

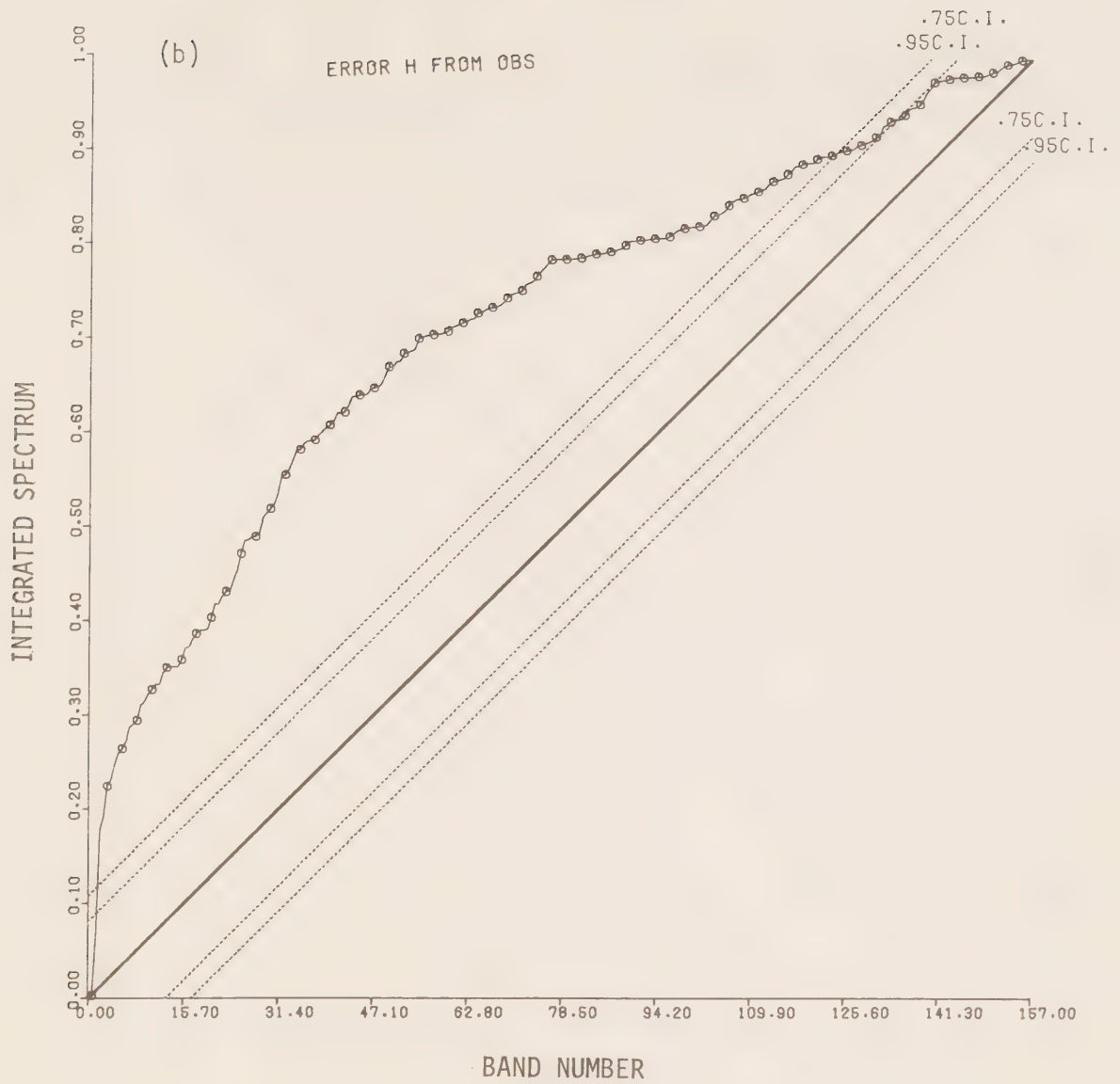


Fig. 3b.

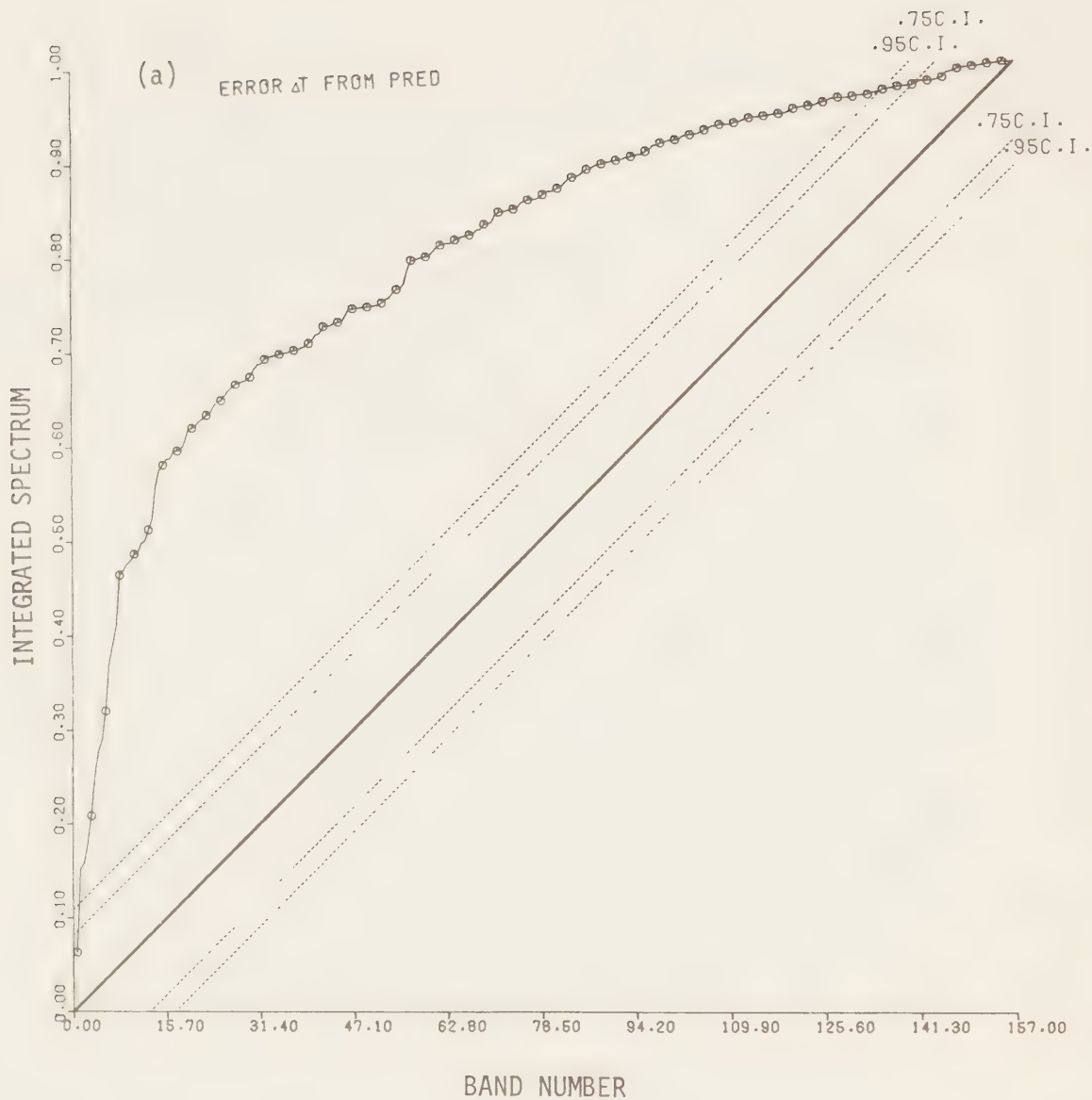


Fig. 4. White noise test on the residues in the time differences. a) From the predicted levels at Saint John. b) From the observed levels at Saint John.

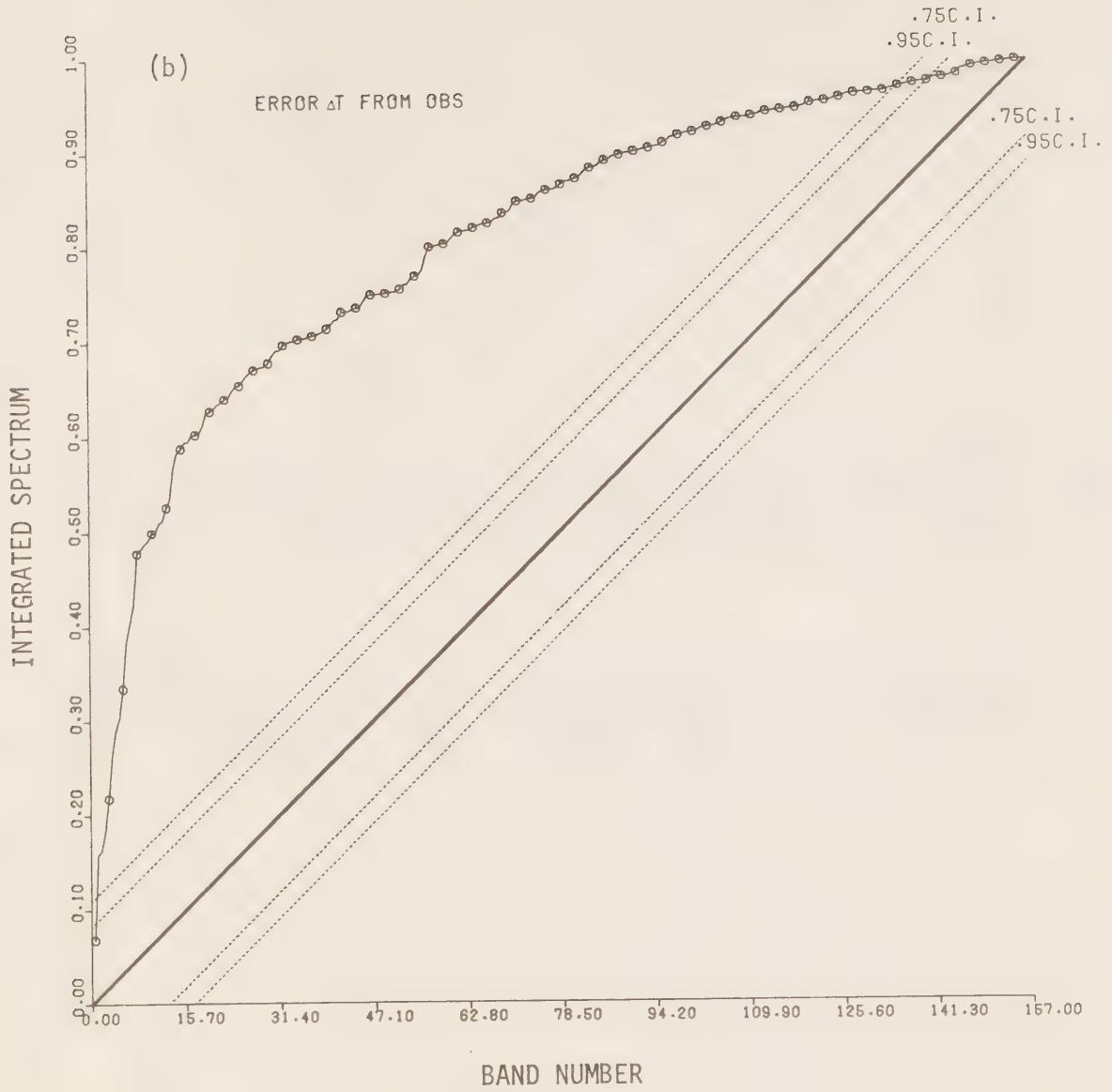


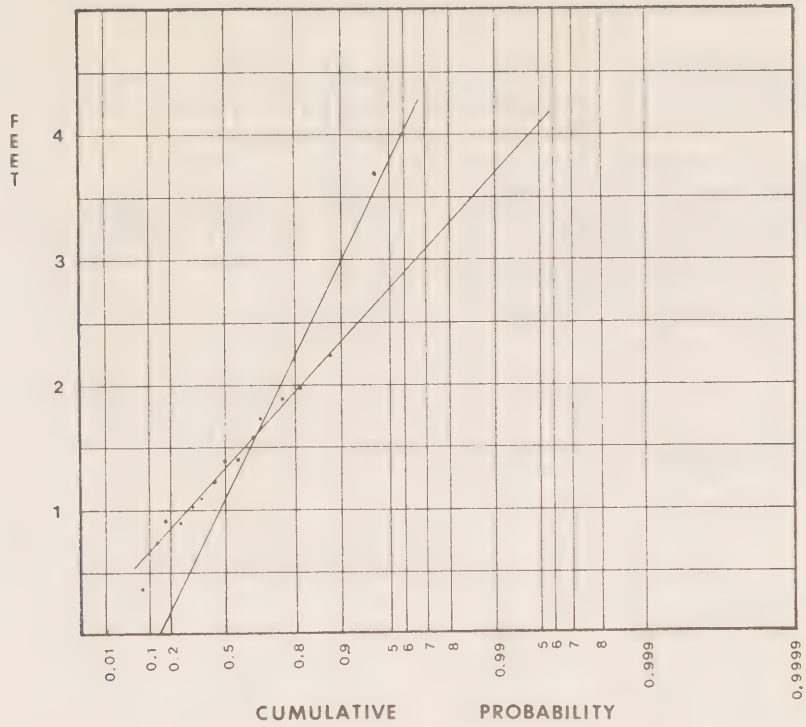
Fig. 4b.

Fig. 5. Monthly extremes in the residues in the predicted height of high water at BLB using a regression with the predicted or observed height of high water at Saint John on probability paper (double exponential distribution).

$$e^{-e^{-\frac{x-A}{B}}}$$

The slope implied by the largest extreme differs significantly from the slope implied by the remaining points (which may contain values less than the true monthly extremes because of gaps in the observations). a) Predicted height of high water at Saint John. b) Observed height at Saint John.

PREDICTED
(a)



RETURN
PERIOD MONTHS



OBSERVED
(b)

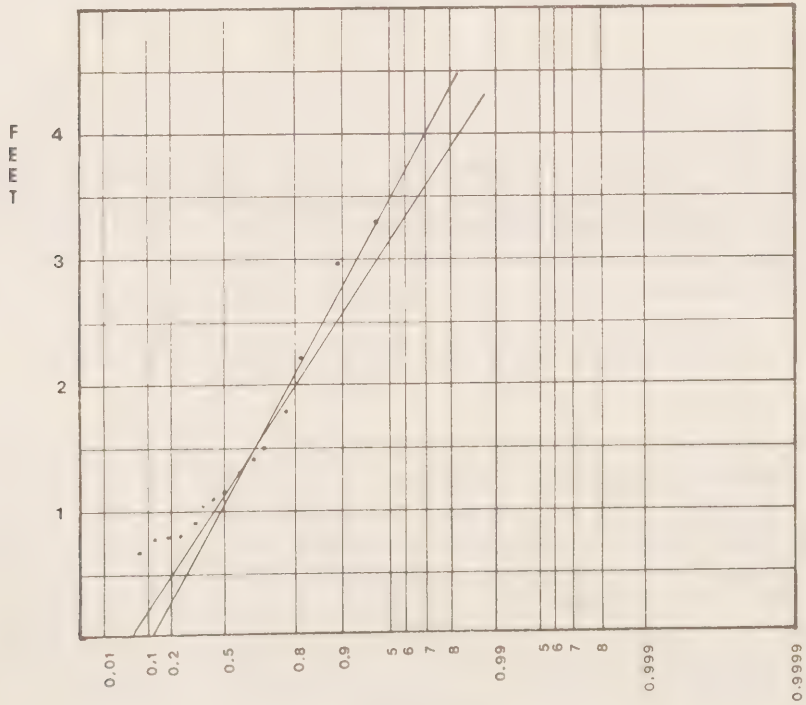


Table 7. Sample monthly extremes in the height residues at the time of high water.

Date	Residues using Observed Water Level at Saint John (ft)	Order	Residues using Predicted Tidal Height at Saint John (ft)	Order
04/71	.69	1	1.38	8
05	1.32	9	1.23	7
06	1.49	11	1.89	12
07	2.22	13	1.99	13
08	1.77	12	2.24	14
09	1.43	10	1.40	9
10	.78	2	.91	4
11	.80	4	.37	1
04/72	2.98	14	1.75	11
05	1.11	7	.76	2
06	.78	3	1.10	6
08	.91	5	.90	3
09	1.16	8	1.02	5
10	3.29	15	3.67	15
11	1.05	6	1.60	10

uncertain. The outlier for the predictions may be due to faulty observations or to large nightly tides which had been missed during the other months because of the intermittent observations during 1971, but we have no way of checking that.

6. RETURN PERIODS AND THE CORRESPONDING HEIGHTS

The return periods in the plots are in months. For Saint John predicted, a 5 month return period falls between 2.0 and 2.2 ft depending on which slope is chosen; for the observed it falls between 2.0 and 2.1 ft. We consider the regression between Saint John and the site as giving the deterministic part of the signal: 29.1 ft at Saint John regresses to 31.1 ft of high water height at BLB for the predicted tide and 31.5 ft for the observed level. To this we add the extreme for a 5 month return period (we stick to low return periods for the residuals because the slope of the extreme is much too uncertain for high values) which is 2.0 to 2.2 ft using the predicted height and

2.0 to 2.1 ft using the observed height. The values are:

$$31.2 + 2.0 \text{ to } 2.2 = 33.1 \text{ to } 33.3 \text{ ft using the predicted height,}$$

and

$$31.5 + 2.0 \text{ to } 2.1 = 33.5 \text{ to } 33.6 \text{ ft using the observed height.}$$

We assign a probability of once in 18 years to the 29.1 ft tide; we call it "assigned" because the tide is not a random signal. We have seen that for Saint John we have two 29.1 ft tides in recent historical time and the 100 year predictions supply four 29.1 ft tides in 100 years, although they are definitely not randomly distributed in time. In this way the compound probability of a 29.1 ft tide and a 2.0 to 2.2 ft residual is once in 90 years and 33.3 ft is the height for the highest high water at BLB with a return period of 90 years for the highest predicted tide at Saint John. The heights deduced from observed levels at Saint John are higher and we feel that their reliability is less.

A return period of 180 years supplies heights of:

$$31.1 + 2.4 \text{ to } 3.0 = 33.5 \text{ to } 34.3 \text{ ft for predicted heights,}$$

and

$$31.5 + 2.5 \text{ to } 2.8 = 34.0 \text{ to } 34.3 \text{ ft for the observed levels.}$$

The search for a return period of 180 years has pushed us further out in the probability curve of the extremes where the slope is very uncertain. We feel it is better to retain 33.3 ft with a return period of 90 years for an extreme height at BLB.

7. SEARCH FOR THE MONTHLY TIDE IN MINAS BASIN

The lack of whiteness of the residues suggests that they still contain a deterministic long-period signal; likely candidates for this source of variability are the semimonthly and the monthly tides which should be present in the upper reaches of the basin because of strong frictional effects. This monthly tide simply cannot be extracted from the observations at BLB, but there is some possibility that it is present in the record of Cobequid. In order to search for it we calculated the cross spectrum between the root mean square of the observed level at Saint John over 25 hours sampled hourly:

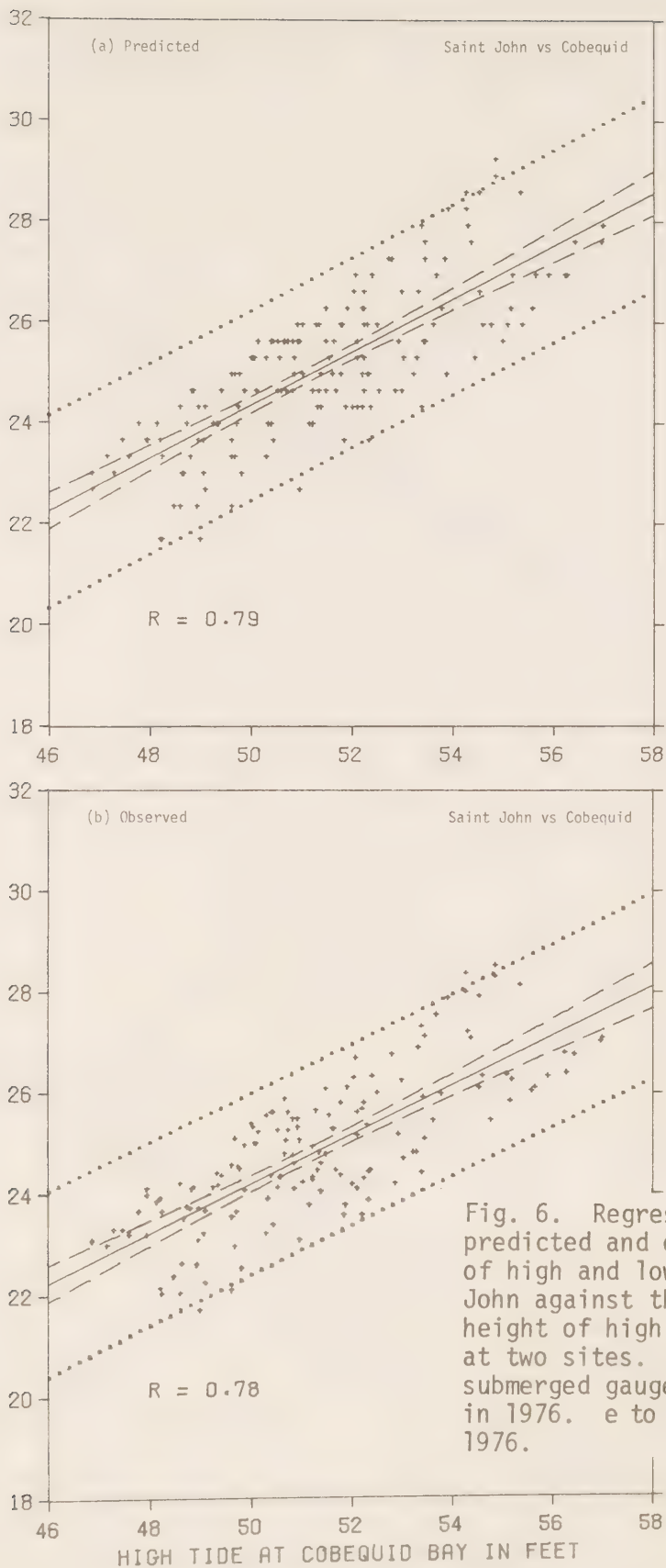
$$\bar{x} = \sqrt{\frac{\sum_i (x_i - x_0)^2}{25}}$$

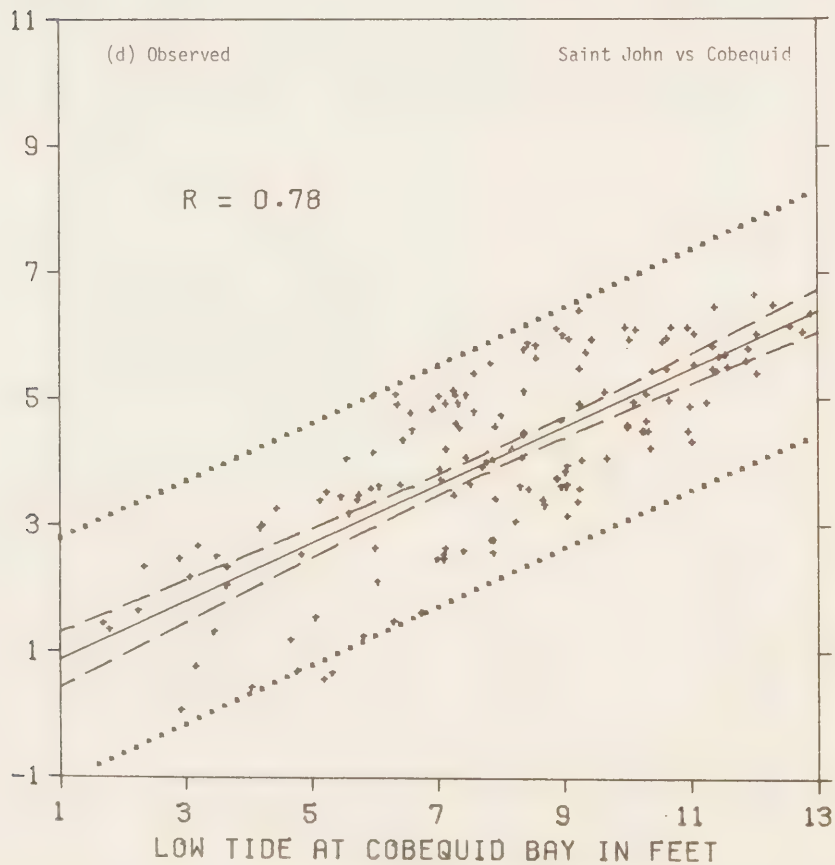
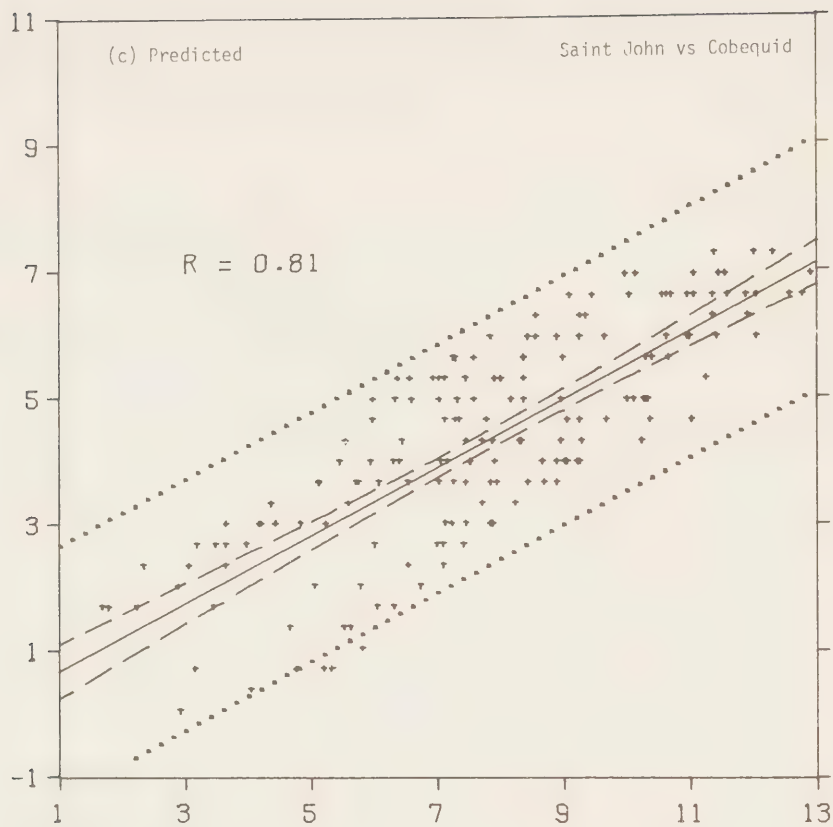
(which models the envelope of the levels at Saint John which drives the monthly tide further upstream (x_i is the observed level, x_0 is the mean level)) and the low pass of the level observed at Minas in 1976. The spectrum of Cobequid does indeed exhibit two marked peaks at the frequencies of $1c/27.9$ d and $1c/13.9$ d and the coherence of the two peaks with the mean square level at Saint John is 0.84 for the first peak and 0.97 for the second peak: they are therefore significant. However the admittance gives amplitudes of

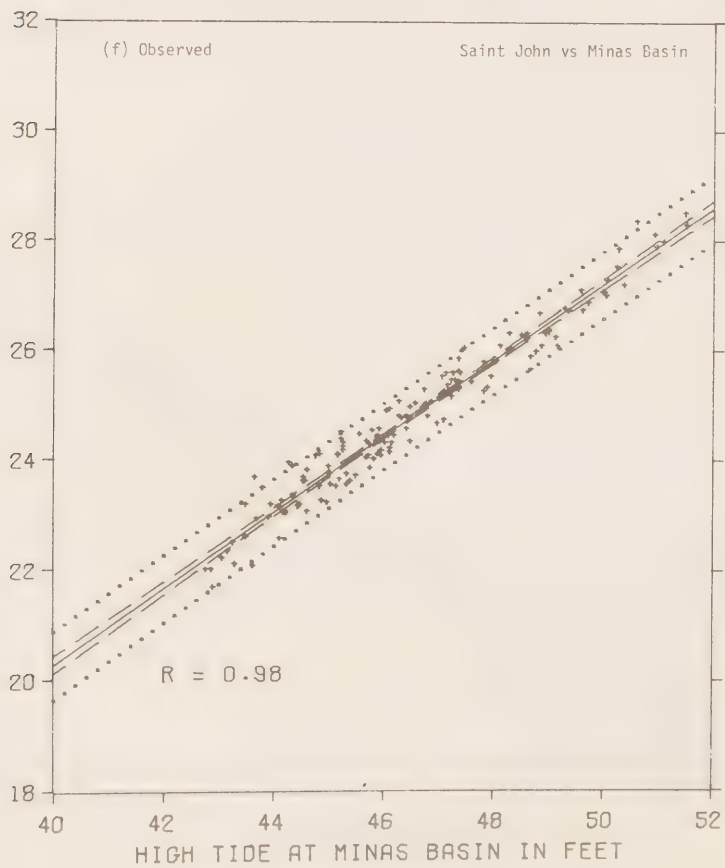
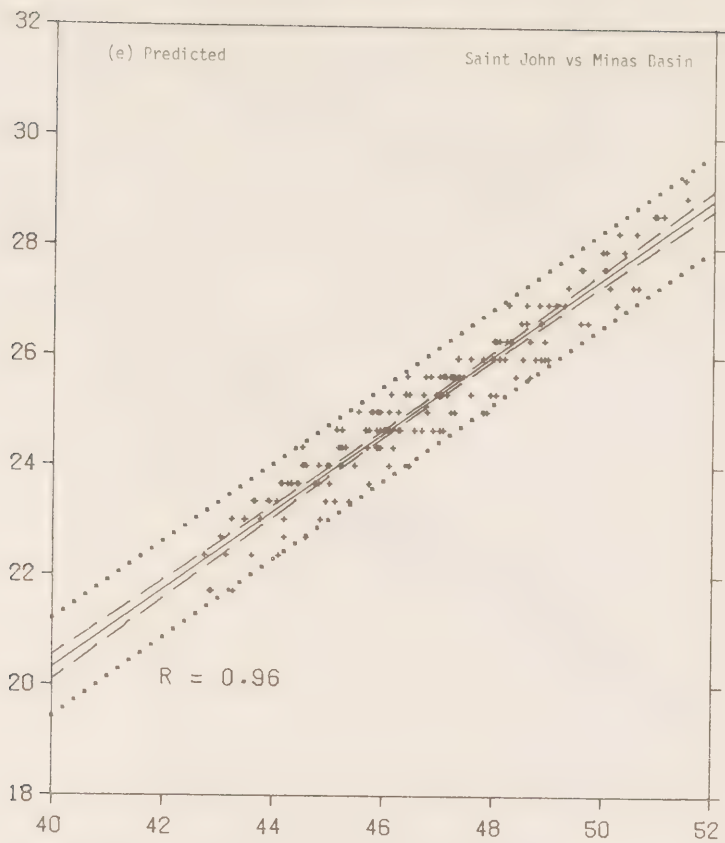
0.05 ft for the monthly signal and 0.06 ft for the semimonthly signal. We therefore have a monthly tide at the submerged gauge site, which is yet too small to be of practical significance, but there remains the possibility that it becomes much stronger at BLB.

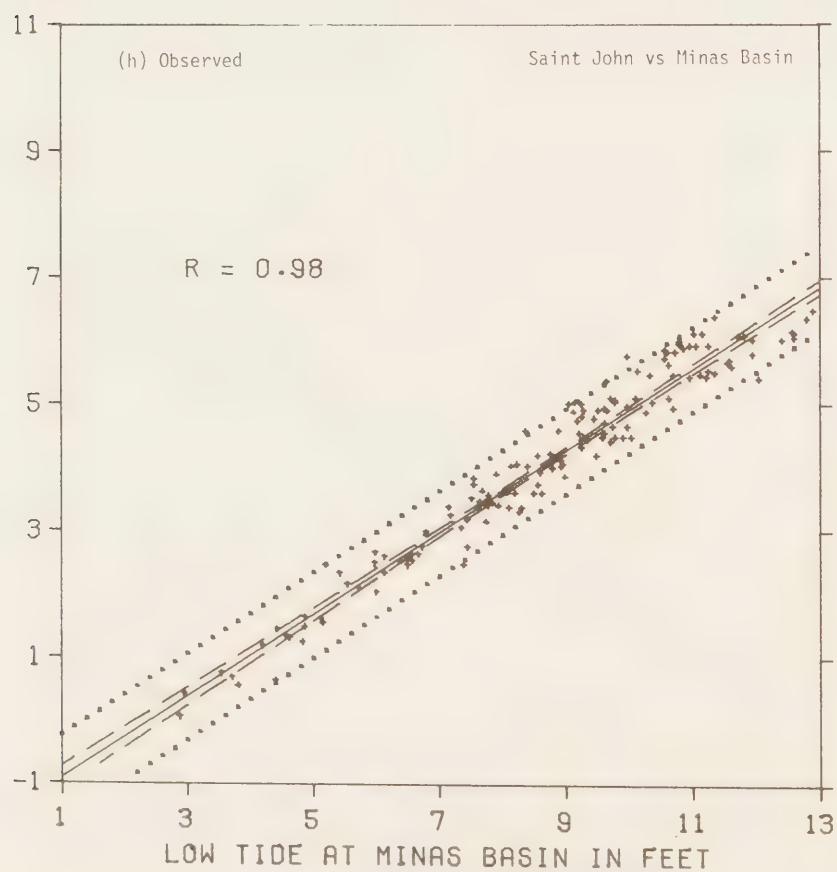
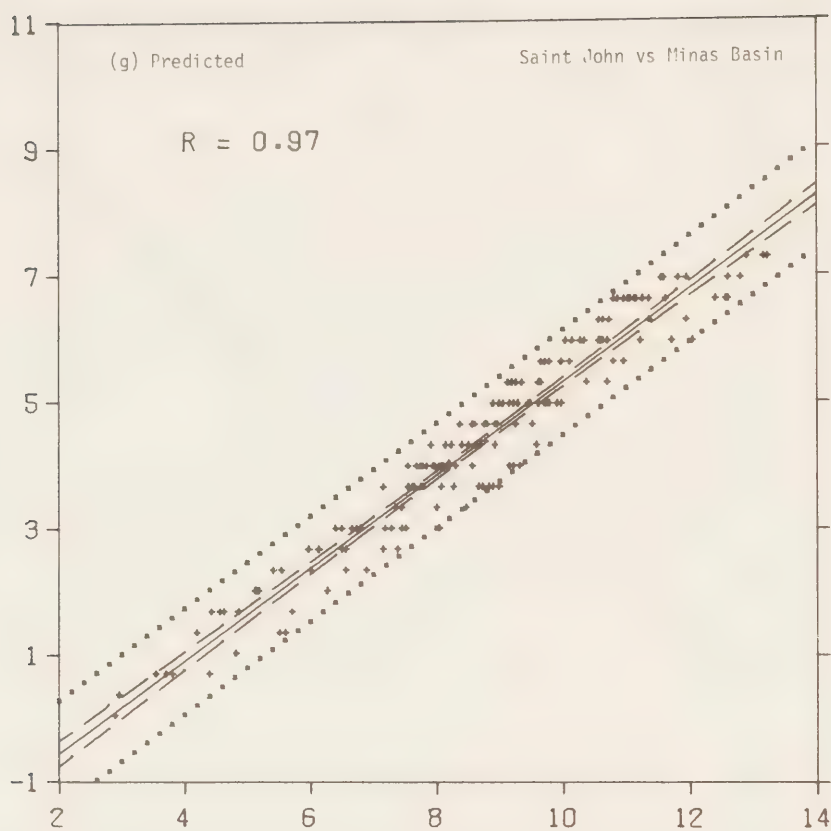
7.1 Regressions between the high and low waters at Cobequid, Minas and Saint John

Regressions were calculated between the actual observations in 1976 on the height and time of high and low water at Minas and Cobequid and the height of high or low water at Saint John (Figures 6 and 7). We note that the regression on the heights at Minas is better with the observed level at Saint John, while at Cobequid the regression is better with the predicted at Saint John, especially low water. This confirms what we have noted at BLB, namely that events in the upper reaches of Minas Basin are probably of local origin and are not strongly coupled with those over the main body of the Bay of Fundy. Also interesting is that the time taken for high water to reach the two submerged sites increases with the height of high water (the higher the high water, the slower the wave), while the reverse holds for the low water. We recall that with respect to Salmon River, the higher the high water at Saint John, the faster it travels upstream. The change in the time of arrival of the wave at the submerged gauge sites for the observed range of high or low water at Saint John is of the order 0.5 to 0.6 h: this is quite minimal but it may help throw some light on the dynamics of the Bay of Fundy-Minas Basin system. We note finally that the scatter in the recorded heights at Cobequid is much larger than at Minas or at BLB; which suggests that some difficulties were encountered in maintaining the calibration of the instrument *in situ* during the observations.









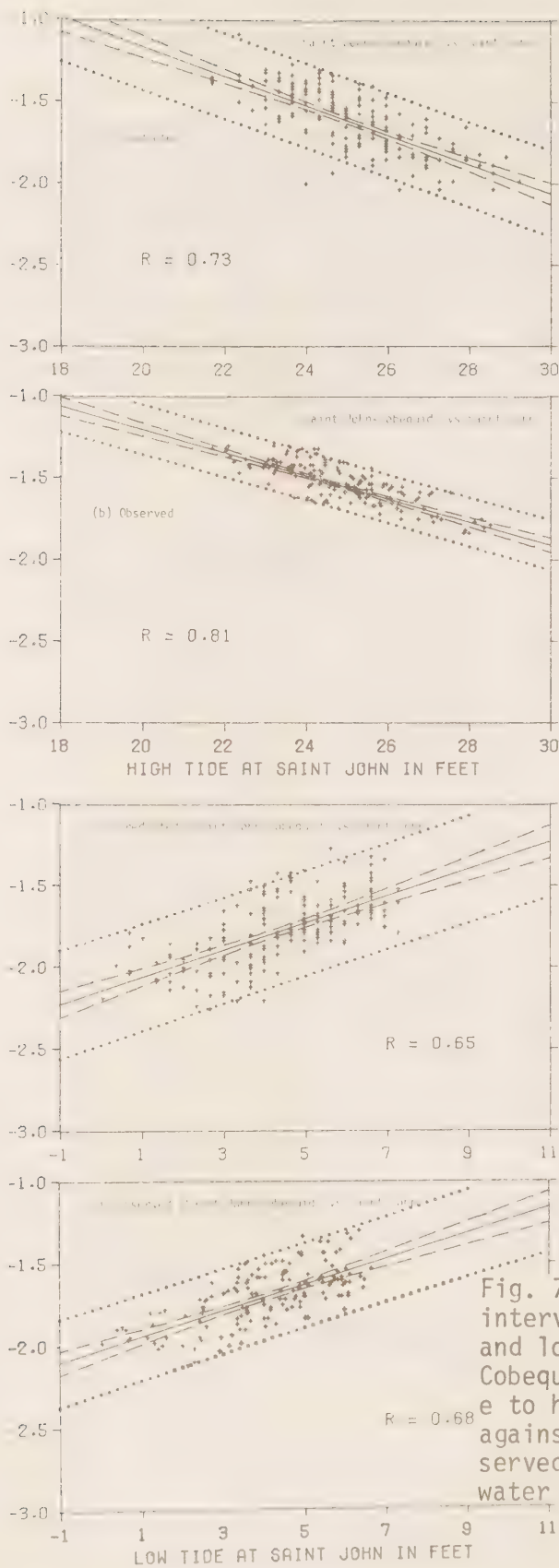
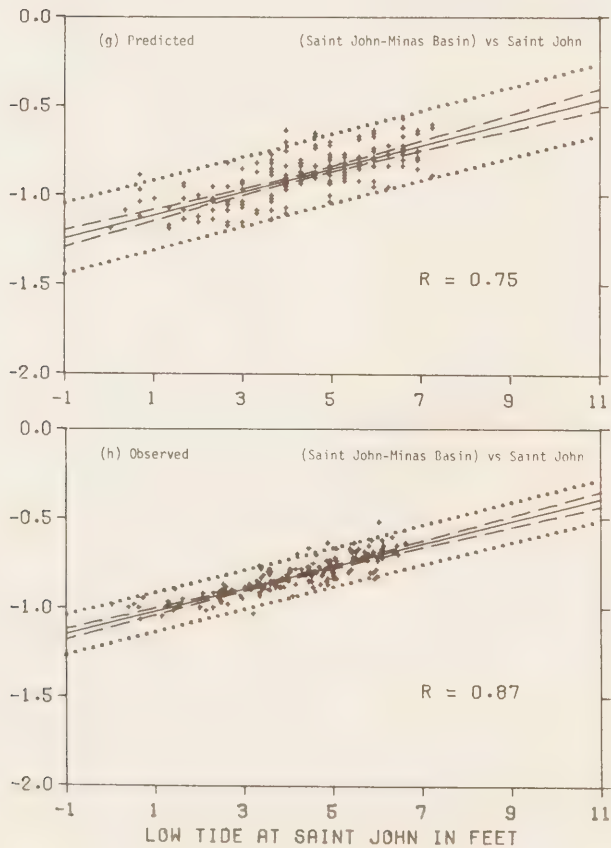
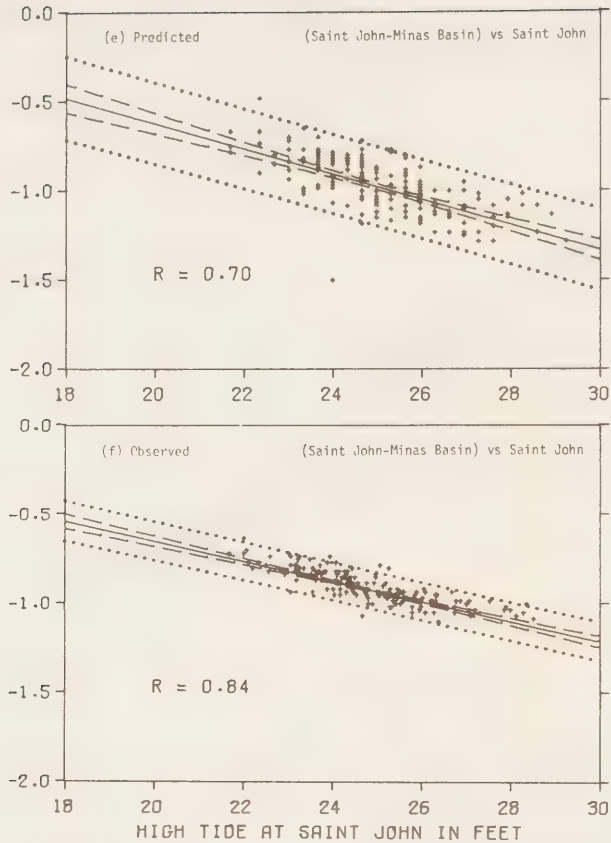


Fig. 7. Regression of the time interval needed for the high and low water to reach a to d) Cobequid from Saint John and e to h) Minas from Saint John against the predicted and observed height of high and low water at Saint John.



8. DISCUSSION

We attempted to obtain an estimate of the extreme level which could be reached at Board Land Bridge due to a combination of the tide and of a disturbance in Minas Basin; we neglected the possibility of a coincident flood (apparently a possibility in the spring) because such considerations lie outside our field of competence. Extreme tides (Table 6) occur mainly in the spring i.e. March, April, May, with fewer occurrences in the fall, i.e. Oct, Nov, Dec, the underlined months having a higher probability of containing an extreme. The spring extremes invariably occur around midnight while the fall extremes occur around noon. The highest tide which reaches a level of 29.1 ft at Saint John, being regressed to 31.1 ft at BLB, does not represent an extremely high level at BLB and what matters most is the possible coincidence of a weather disturbance being superimposed. The maximum value of the disturbance is not related to the tide and it may have its peak at any time before or after high water. But at BLB only high water reaches the site and we must restrict our considerations to the coincidence of high water with a given value of the disturbance. The sample of extreme residues between the value of the level at BLB regressed from Saint John and the actual level observed is taken as a measure of the disturbance present in Minas Basin at the instant of high water; it cannot be a measure of the peak value of the individual disturbance since the site chosen makes it impossible to follow the full development of surges in the basin. Like most sample of extremes it is not easy to interpret, especially because of the value of 3.67 ft which it contains for 1972; (Table 7) the point falls well off the regression curve for the other samples (Fig. 5a) and we simply cannot decide if we should keep it or reject it. Our solution to the dilemma is to pick an extreme of low probability (1/5 months) whose position is very little affected by this outlier and calculate its compound probability with the rarest tidal event. In order to understand the influence of tide heights on the probability of occurrence, we did note that the tide has an upper bound in contrast to a random variable. In the case of Saint John, the upper bound is slightly over 29.1 ft, let us say 29.2 ft. We have an extremely rapid decrease in assigned probability going from a spring tide at 26.1 ft (26.4 ft at BLB) twice a month, to a perigean tide of 26.6 ft (27.2 ft at BLB) once a month, to a spring perigean tide of 28.3 ft (29.7 ft at BLB) once every 13 months, to a declinational spring perigean tide of 29.1 ft (31.1 ft at BLB) once every 19 years to the upper bound of 29.2 ft (31.2 ft at BLB) which has a probability of near zero. It is easy to pick a tide level with an assigned value of probability. We definitely cannot do the same thing with the weather disturbances because of the shortness of the observations and the selection of the site of measurements: we did the best we could with what was available. The highest level we selected is one of the least probable although the level of probability we quoted (once in 90 years) cannot be considered as absolute in view of the rapid change in the assigned probability in the region of extreme tides. High levels with a higher probability could have been selected such as:

- 1) a perigean tide of 27.2 ft (at BLB) once a month with a disturbance of 2.2 ft (once per 8 months)

$$27.2 + 2.2 = 29.4 \text{ ft once per 8 months, or}$$

- 2) a spring perigean tide of 29.7 ft (at BLB) with a disturbance of 2.2 ft

$$29.7 + 2.2 = 31.9 \text{ ft once every nine years.}$$

These extreme levels are lower than the one quoted in section 6 and can be reached more often. We were reluctant to use the extreme disturbance of 2.67 ft as it does not seem to belong. For a spring perigean tide of 29.7 ft in conjunction with such an extreme, we would have:

$$29.7 + 2.6 \rightarrow 3.7 = 32.4 \rightarrow 33.4 \text{ ft once every 16 years.}$$

The margin of 1.1 ft implied by the uncertainty in the fitting curve to the probability distribution makes this estimate too blunt to use.

We conclude that we have gone as far as we could in the study of the meagre set of data presently at our disposal. Some serious studies on the dynamics of Minas Basin will eventually have to be undertaken. To a good first approximation the basin may be viewed as cut off from the Bay of Fundy as far as disturbances are concerned and the first step in such a study could be the installation of a gauge capable of measuring the full range of the tide over at least 2 or 3 years with in addition, a network of a few additional gauges at other sites over a shorter interval.

9. ACKNOWLEDGEMENTS

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